Int. J. Solids Structures Vol. 21, No. 4, pp. 325–342, 1985
Printed in the U.S.A.

0020-7683/85 $3.00 + .00

A PREDICTION OF HEAT GENERATION IN A THERMOVISCOPLASTIC UNIAXIAL BAR

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(*Received* 25 *July* 1983)

Abstract—A thermodynamic model is presented for predicting the thermomechanical response, including temperature change, in a uniaxial bar composed of a thermoviscoplastic metallic medium. The model is constructed using the concept of internal state variables, and it is shown that this general framework is capable of encompassing several constitutive models currently used to predict the response of rate-sensitive metals in the inelastic range. Results are obtained for monotonic loading which agree with predicted results previously obtained by Cernocky and Krempl for mild steel at room temperature. The model is then utilized in conjunction with Bodner and Partom's constitutive equations to predict temperature change in Inconel (IN) 100 subjected to both monotonic and cyclic loading at 1005 K (1350°F).

INTRODUCTION

It has long been known that mechanical and thermodynamic coupling exists in solid bodies[1, 2]. However, in elastic bodies this coupling is negligible except when mass inertia is not negligible due to flux of heat generated through the boundary of the body[3]. However, in thermoviscoplastic metals the conversion of mechanical energy to heat may be significant even under non-inertial conditions, especially since material properties become extremely temperature sensitive in the inelastic range of response[4–11]. Similar research has been performed on non-metallic media[12–15].

General continuum mechanics models have been formulated for broad classes of materials[16–19]. However, to this author's knowledge only recently has attention been paid to the coupled heat-conduction equation for thermoviscoplastic metals[11, 20]. Recently, Cernocky and Krempl[11] proposed a model for predicting the temperature rise in a class of thermoviscoplastic metals, with special emphasis on test coupons subjected to either homogeneous uniaxial or torsion loadings. In this paper an alternative approach to that proposed in [11] is discussed. This method uses the thermodynamics with internal-state variables originally reported in [17] and discussed elsewhere in detail for metals[18, 21, 22] with development of the multidimensional coupled heat-conduction equation in [20].

The research herein is presented in three parts: field formulation in one-dimensional form; development of the governing equations from additional constitutive assumptions; and numerical results for selected problems.

THERMODYNAMICS OF A UNIAXIAL THERMOVISCOPLASTIC BAR

Consider a slender bar which is subjected to a homogeneously applied deformation field such that the resulting stress field is everywhere uniaxial in the $x_1 \equiv x$ coordinate direction, as shown in Fig. 1. Rigor would require that the possibility of finite deformations be considered. However this condition is covered in detail elsewhere[17, 18, 20–22], and for purposes of simplicity only infinitesimal deformations will be considered herein. For notational simplicity, then, the observable mechanical state variables are

$$u \equiv u_1 = \text{deformation field,} \tag{1}$$

$$\epsilon \equiv \epsilon_{11} = \text{infinitesimal strain field and} \tag{2}$$

$$\sigma \equiv \sigma_{11} = \text{stress field.} \tag{3}$$

Although transverse components of deformation and strain may occur, it is assumed that they are not necessary to characterize the unaxial stress σ.

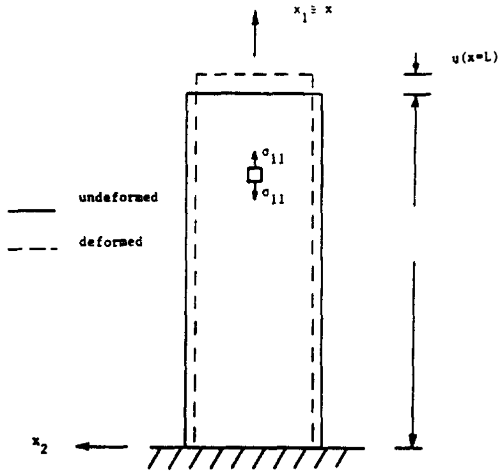

Fig. 1. Geometry and deformations in a uniaxial bar.

The mechanical state variables (1)–(3) are adjoined with the thermodynamic state variables:

$$e \equiv \text{internal energy per unit mass;} \tag{4}$$

$$r \equiv \text{heat supply per unit mass;} \tag{5}$$

$$s \equiv \text{entropy per unit mass;} \tag{6}$$

$$T \equiv \text{absolute temperature and} \tag{7}$$

$$q \equiv q_1 = \text{heat flux in the } x_1 \text{ coordinate direction,} \tag{8}$$

where it is assumed in (8) that the bar is isotropic and long and slender with perfectly longitudinal boundaries so that the heat flux is one-dimensional.

In accordance with the theory of internal state variables[17], observable state variables (1)–(8) are now supplemented with internal state variable growth laws in order to characterize the state of inelastic bodies:

$$\alpha_k \equiv \text{scalar valued internal state variables, } k = 1 \ldots n; \tag{9}$$

where n is the number of internal state variables required to characterize the state of the body. The precise nature of (9) will be discussed later.

Parameters (1)–(9) are assumed to be functions of space (x) and time (t), and are assumed to be sufficient to describe the uniaxial state of the bar at all times. These parameters are constrained by

(a) the conservation of momentum,

$$\frac{\partial \sigma}{\partial x} = 0, \tag{10}$$

where inertial effects and the body force are assumed to be negligible;

(b) the strain-displacement relation,

$$\epsilon = \frac{\partial u}{\partial x}; \tag{11}$$

(c) the balance of energy,

$$\rho\dot{e} - \sigma\dot{\epsilon} + \frac{\partial q}{\partial x} = \rho r, \tag{12}$$

where ρ represents the mass density; and

(d) the second law of thermodynamics,

$$\rho\gamma \equiv \rho\dot{s} - \frac{\rho r}{T} + \frac{\partial}{\partial x}\left(\frac{q}{T}\right) \geq 0, \tag{13}$$

where γ is called the internal entropy rate per unit mass.

As detailed by Coleman & Gurtin[17], eqns (10)–(13) are now supplemented with the following constitutive assumptions:

$$\sigma = \sigma(\epsilon, T, \partial T/\partial x, \alpha_k); \tag{14}$$

$$e = e(\epsilon, T, \partial T/\partial x, \alpha_k); \tag{15}$$

$$s = s(\epsilon, T, \partial T/\partial x, \alpha_k); \tag{16}$$

$$q = q(\epsilon, T, \partial T/\partial x, \alpha_k) \text{ and} \tag{17}$$

$$\dot{\alpha}_k = \dot{\alpha}_k(\epsilon, T, \partial T/\partial x, \alpha_k), \tag{18}$$

where it is obvious that eqns (14)–(18) satisfy the principle of equipresence[23]. Equations (10)–(12) and (14)–(18) describe eight $+n$ equations in the eight $+n$ field variables u, ϵ, σ, e, r, s, T, q, and α_k described in (1)–(9). These are adjoined with boundary conditions on the surfaces $x = 0$ and $x = L$ to prescribe the one-dimensional field problem.

As detailed elsewhere[17, 18, 20–22], the second law of thermodynamics [inequality (13)] will constrain constitutive assumptions (14)–(18). This is accomplished by defining the Helmholtz free energy:

$$h \equiv h(\epsilon, T, \partial T/\partial x, \alpha_k) = e - Ts \Rightarrow e = h + Ts, \tag{19}$$

which together with the Clausius-Duhem inequality will lead to the conclusions that

$$h = h(\epsilon, T, \alpha_k), \tag{20}$$

$$s = \frac{-\partial h}{\partial T} = s(\epsilon, T, \alpha_k), \tag{21}$$

$$\sigma = \rho\frac{\partial h}{\partial \epsilon} \quad \text{and} \tag{22}$$

$$q = -k\frac{\partial T}{\partial x_1} + 0\left|\frac{\partial T}{\partial x_1}\right|, \tag{23}$$

where k is the coefficient of heat conduction in the x_1-coordinate direction. Therefore, eqns (19)–(23) replace eqns (14)–(18) as more concise statements of the constitutive behavior, and it can be seen that specification of the Helmholtz free energy will complete the description of the field problem.

Combination of eqns (12) and (19)–(23) will result in the coupled heat conduction equation:

$$\rho\frac{\partial h}{\partial \alpha_k}\dot{\alpha}_k - \rho T\frac{\partial^2 h}{\partial \alpha_k \partial T}\dot{\alpha}_k - \rho T\frac{\partial^2 h}{\partial \epsilon \partial T}\dot{\epsilon} - \rho T\frac{\partial^2 h}{\partial T^2}\dot{T} + \frac{\partial q}{\partial x} = \rho r, \tag{24}$$

where summation on the range of k is implied.

Henceforth in this investigation it will be assumed that there is no internal heat source (other than material dissipation) so that $r = 0$ in eqn (24). In addition, it will be assumed that boundary conditions are applied in such a way that heat flux is negligibly small and $q \cong 0$ in eqn (24). This last assumption is not valid under most physical circumstances. However, it can be said that, on the basis of heat-conduction eqn (24), neglecting heat flux will result in an upper bound for the temperature rise during mechanically induced energy dissipation. Inclusion of this term results in a spacially dependent boundary value problem which is beyond the scope of the current research. However, the one-dimensional model proposed herein does encompass the heat flux phenomenon, and, as such, will be the subject of a future article by the author.

DEVELOPMENT OF GOVERNING EQUATIONS FROM ADDITIONAL CONSTITUTIVE ASSUMPTIONS

In order to construct the Helmholtz free energy function the elastic strain is first defined to be

$$\epsilon^E \equiv \epsilon - \alpha_1 - \bar{\alpha}\theta, \tag{25}$$

where α_1 is the total inelastic strain in the x_1-coordinate system[24], $\bar{\alpha}$ is the coefficient of thermal expansion in the x_1-coordinate direction, and $\theta \equiv T - T_R$, where T_R is the initial temperature at which no strain is observed under zero mechanical load. The inelastic strain α_1 will be discussed in greater detail in the next section.

It is now postulated that the Helmholtz free energy may be expanded about the initial configuration in terms of elastic strain and temperature as follows:

$$h = h_R + \frac{E}{2\rho}\epsilon^{E^2} - \frac{C_v}{2T}\theta^2, \tag{26}$$

where the subscript R denotes the equilibrium value, and

$$h_R \equiv \text{free energy in state } R = \text{constant}, \tag{27}$$

$$E \equiv \text{Young's modulus in the } x_1\text{-coordinate system}, \tag{28}$$

$$C_v \equiv -T(\partial^2 h/\partial T^2) = \text{specific heat at constant volume}. \tag{29}$$

Note that although the first-order terms in ϵ^E and θ have been neglected due to symmetry conditions due to the form of eqn (25), coupling is retained between total strain, inelastic strain and temperature. Note also that the energy dissipation due to microstructural change has been neglected in free energy equation (26) because this mechanism has been shown to contribute only a small portion of energy (<10%) to the dissipation process [25]. Further, the fracture energy loss due to microvoid growth, grain boundary sliding, and intergranular macrofracture is neglected due to the small strains considered herein.

Although the second-order Taylor series expansion of the Helmholtz free energy given in eqn (26) may not be adequate for characterizing the response of many materials, it will be shown in the next section that the above equations are a suitable framework for describing the material behavior of the class of materials considered herein.

Substitution of eqn (26) into energy balance law (24) and utilizing eqn (25) will result in the coupled heat equation:

$$[(E\epsilon - E\alpha_1 + E\alpha T_R)\dot{\alpha}_1 + E\bar{\alpha}^2 T\dot{T}] - E\bar{\alpha}T\dot{\epsilon} - \rho C_v\dot{T} = 0, \tag{30}$$

where the terms in brackets arise due to inelastic response and the following term is the classical elastic coupling term[3]. Equation (30) may be written in the following equivalent form:

$$\dot{T} = \frac{(E\epsilon - E\alpha_1 + E\bar{\alpha}T_R)\dot{\alpha}_1 - E\bar{\alpha}T\dot{\epsilon}}{\rho C_v - E\bar{\alpha}^2 T} . \tag{31}$$

In order to obtain the stress-strain relation the Helmholtz free energy equation (26) may be substituted into eqn (22) to obtain

$$\sigma = E(\epsilon - \alpha_1 - \bar{\alpha}\theta) \tag{32}$$

Equations (31) and (32), together with internal state variable growth laws (18), will be sufficient to characterize the response of the uniaxial bar subjected to uniaxial homogeneous mechanical loading considered herein.

SELECTED PROBLEMS AND NUMERICAL RESULTS

It has been shown that stress-strain relation (32), together with internal state variable growth laws (18), are equivalent to several models recently proposed for thermoviscoplastic metals[24]. These include Cernocky and Krempl[11, 26], Valanis[27], Krieg *et al.*[28], and Allen and Haisler[29]. It can also be shown that several others are in accordance with the model developed herein[20–34]. To illustrate this point two models have been selected for further discussion.

Cernocky and Krempl's stress-strain relation may be written in the following uniaxial form:

$$\sigma + K(\sigma, \epsilon, T)\dot{\sigma} = G(\epsilon, T) + M(\sigma, \epsilon, T)[\dot{\epsilon} - \bar{\alpha}\dot{T}], \tag{33}$$

where

$$E \equiv M/K \tag{34}$$

and parentheses imply dependence on the current values of the quantities enclosed. Equations (33) and (34) can be shown to be in agreement with stress-strain equation (32) by defining the inelastic strain α_1 such that

$$\dot{\alpha}_1 = [\sigma - G(\epsilon, T)]/M(\sigma, \epsilon, T), \tag{35}$$

so that

$$\alpha_1(t) = \int_{t_R}^{t} \dot{\alpha}_1(t')\, dt', \tag{36}$$

where t_R is the reference time, t' is a dummy variable of integration, and t is the time of interest. Thus, since G, K and M are not history dependent, Cernocky and Krempl's model is a single internal state variable model and eqns (31), (32), (35) and (36) describe the uniaxial bar problem using Cernocky and Krempl's model.

To illustrate this point an example problem is now considered. Several uniaxial bars composed of mild steel are subjected to constant strain rates at room temperature with material properties as described in Table 2 of [11]. Stress-strain behavior and resulting temperature rise are shown in Figs. 2 and 3. These results were obtained by integrating eqns (31) and (35) with a stable and accurate Euler forward integration scheme. Due to the rate-insensitive nature of mild steel at room temperature, the predicted results are identical for strain rates ranging from 0.001 $\sec^{-1}$ to 1.0 $\sec^{-1}$. The negligence of heat flux over such a wide range of strain rates is valid only under adiabatic conditions.

It is significant to note that the results obtained in Figs. 2 and 3 are identical to those

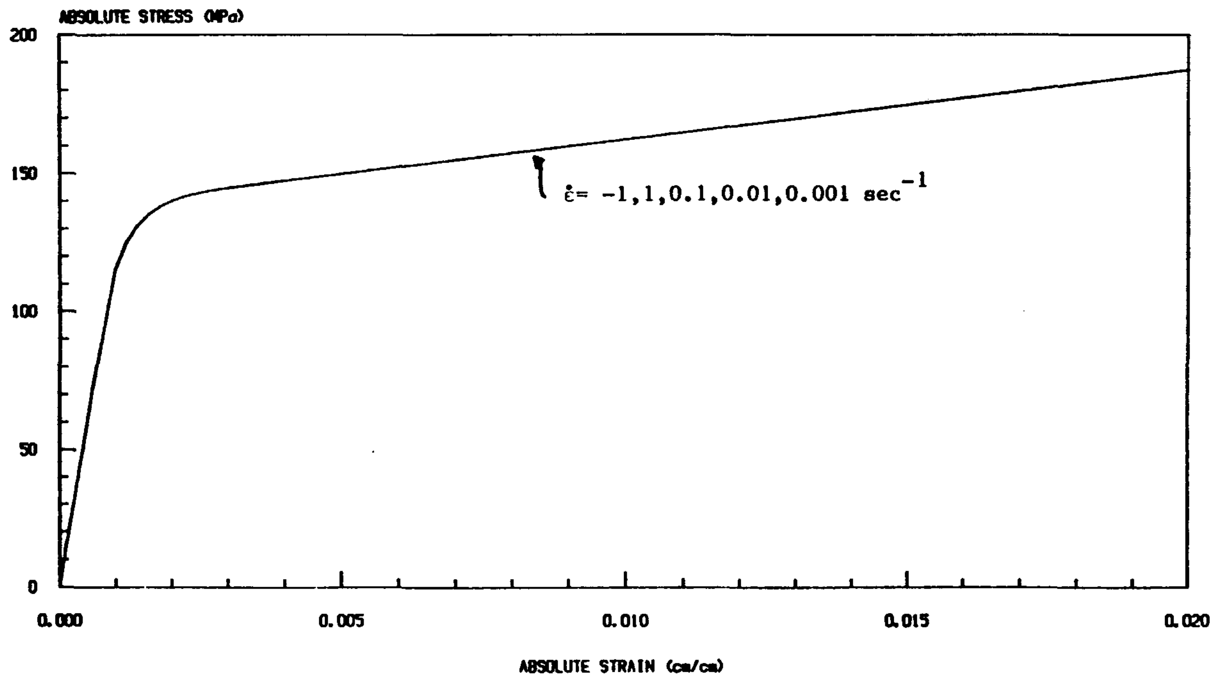

Fig. 2. Predicted stress-strain behavior for mild steel at room temperature subjected to constant strain rate in tension and compression.

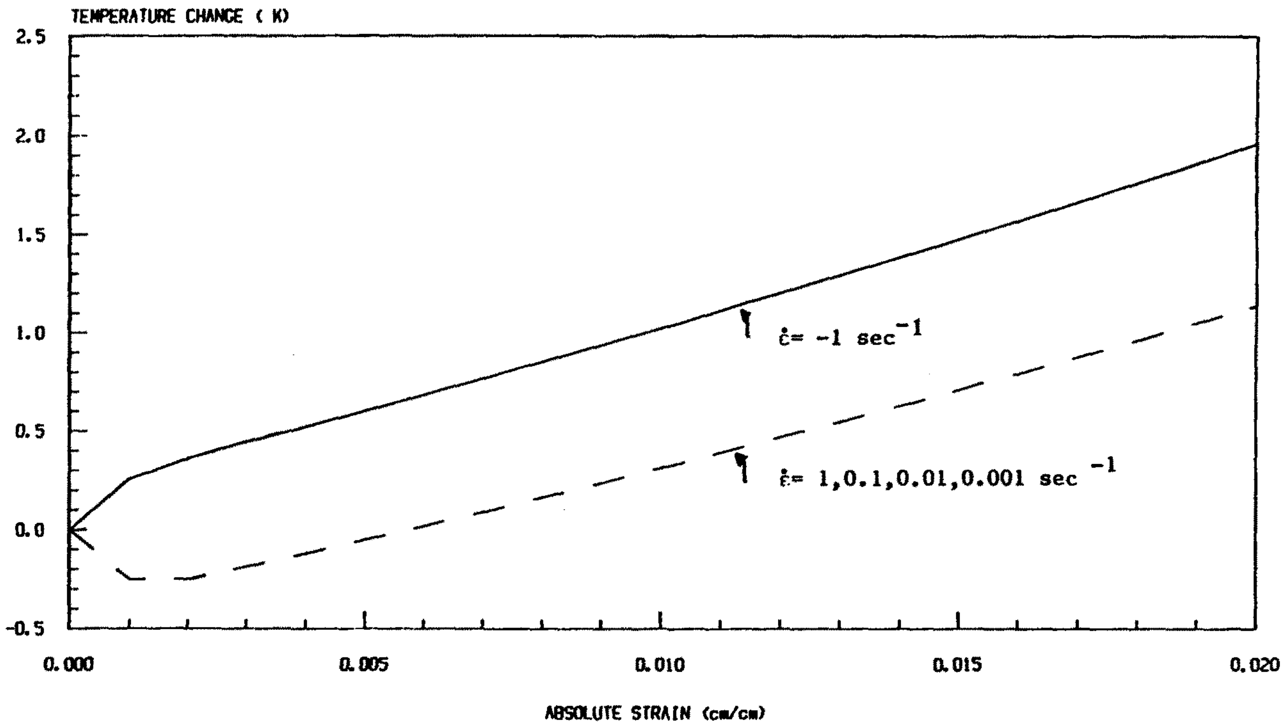

Fig. 3. Predicted temperature change for mild steel at room temperature subjected to load histories shown in Fig. 2.

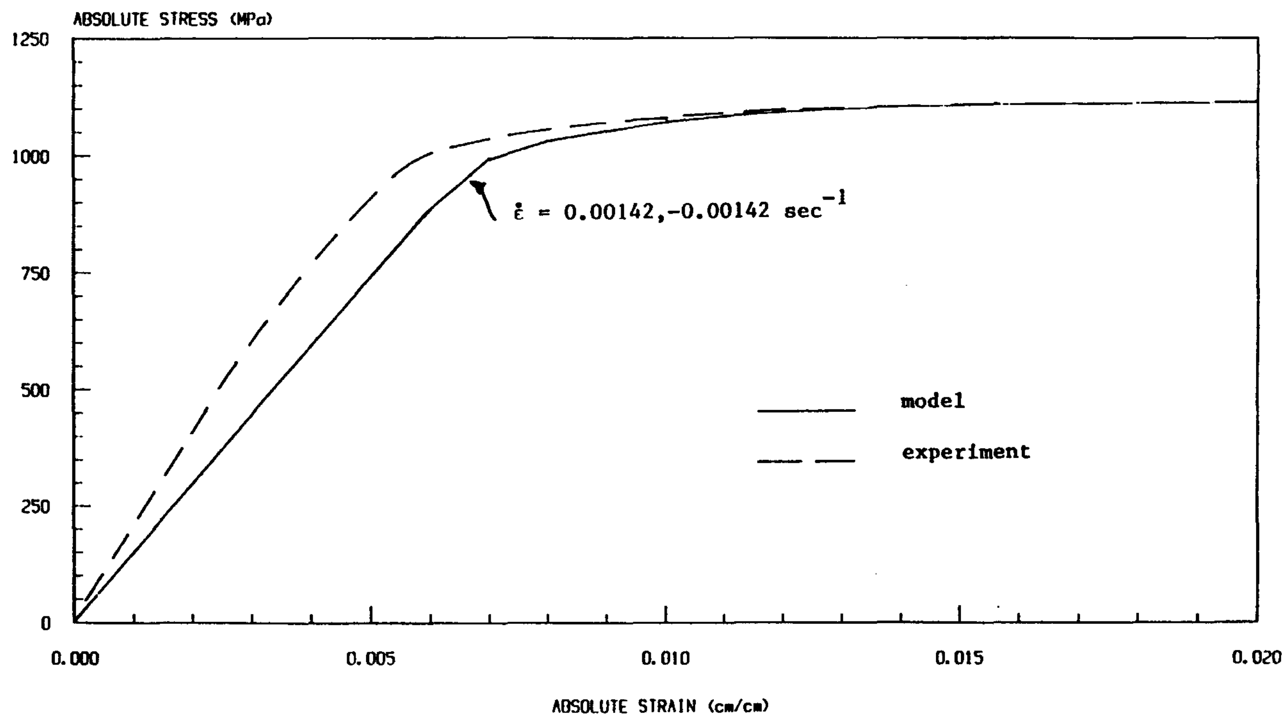

Fig. 4. Stress-strain behavior for IN100 at 1005 K (1350°F) subjected to constant strain rate in tension and compression.

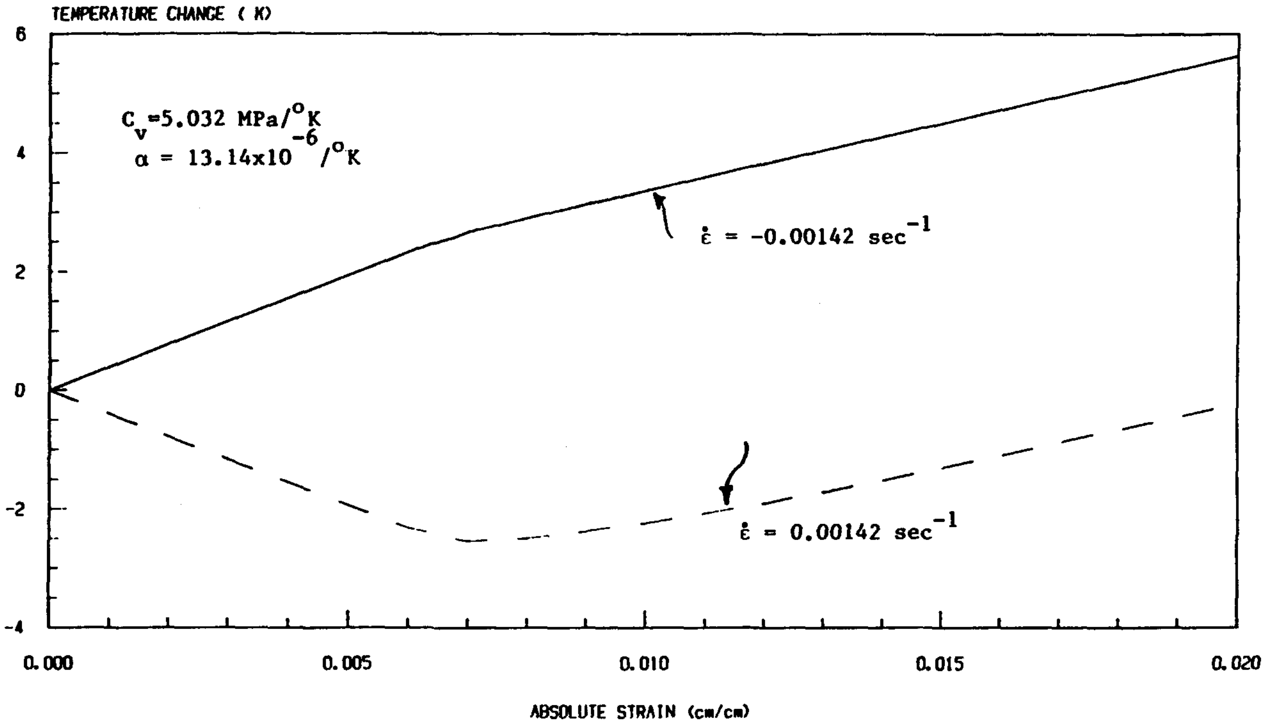

Fig. 5. Predicted temperature change for IN100 at 1005 K (1350°F) subjected to load histories shown in Fig. 4.

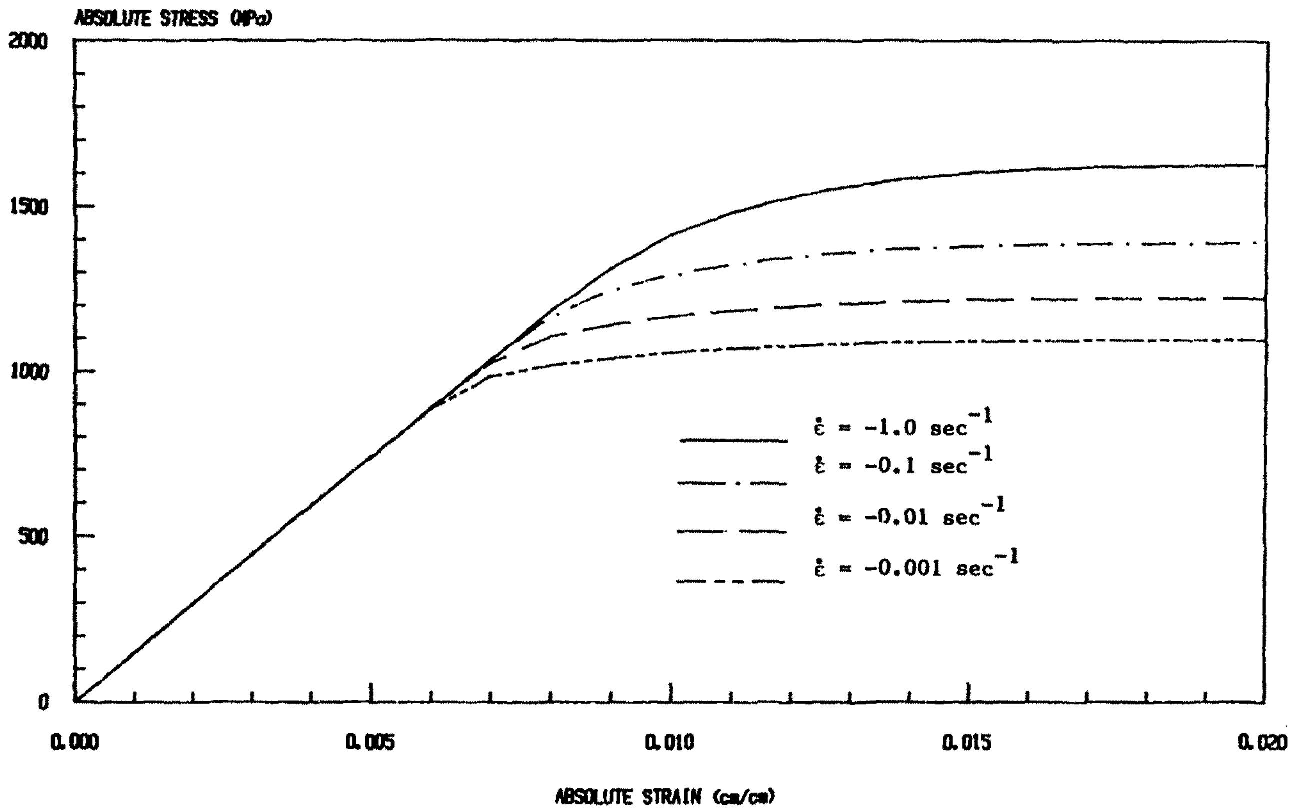

Fig. 6. Predicted stress-strain behavior for IN100 at 1005 K (1350°F) subjected to constant strain rates in compression.

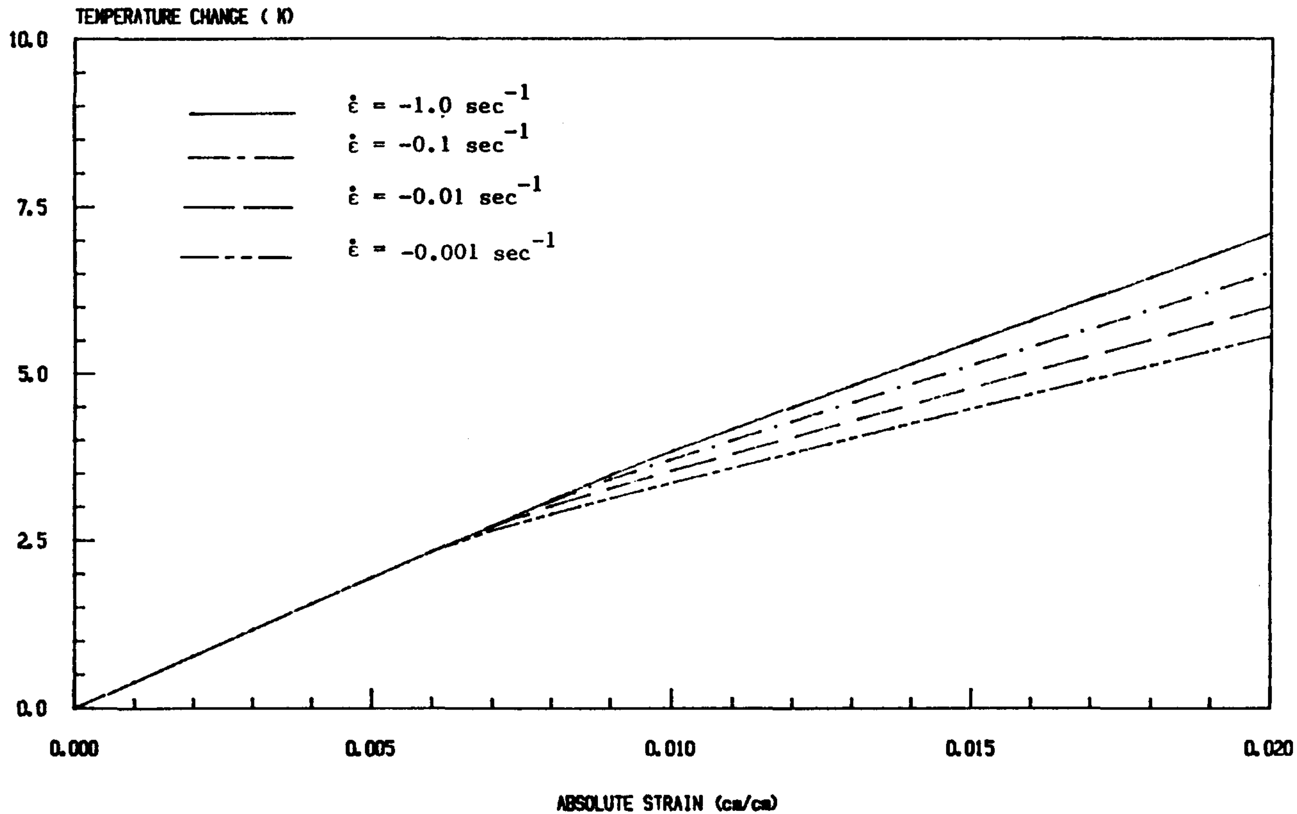

Fig. 7. Predicted temperature change for IN100 at 1005 K (1350°F) subjected to load histories shown in Fig. 6.

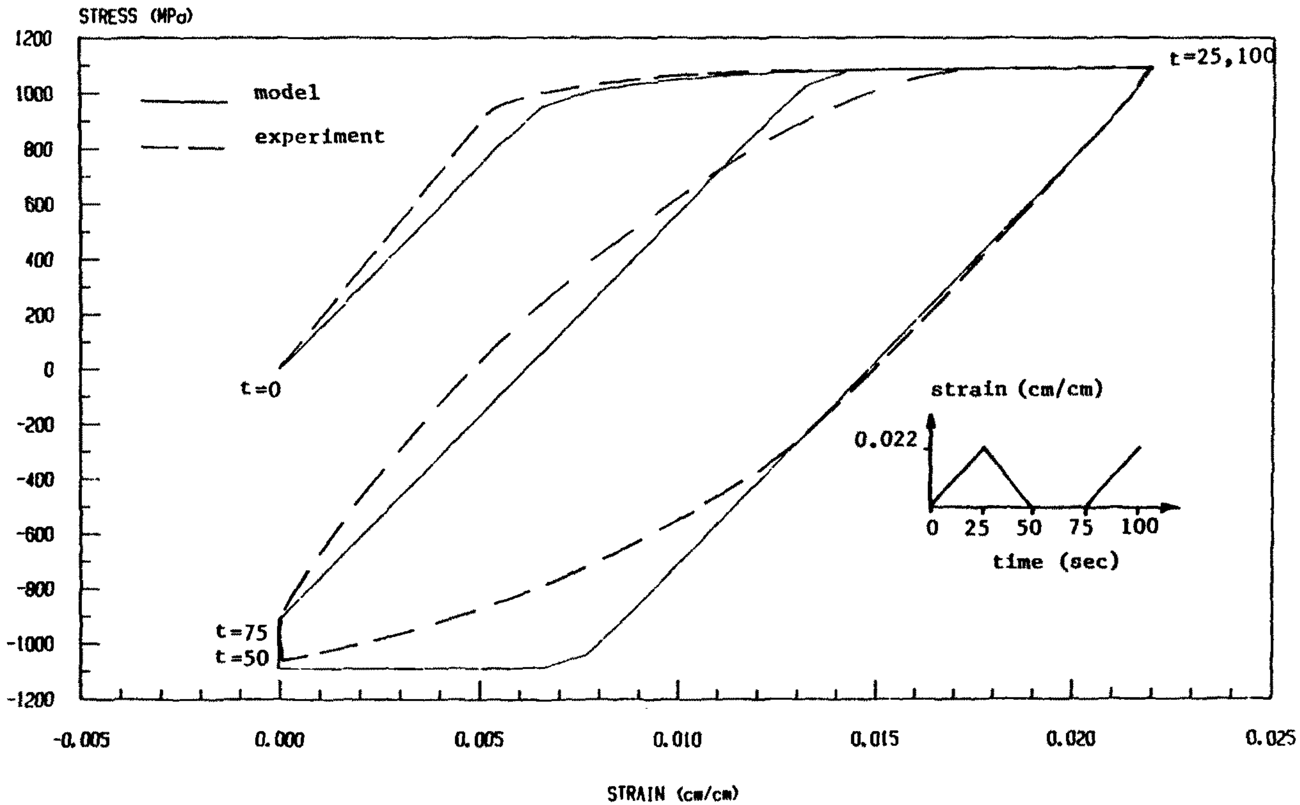

Fig. 8. Stress-strain behavior of IN100 at 1005 K (1350°F) under cyclic load with stress relaxation.

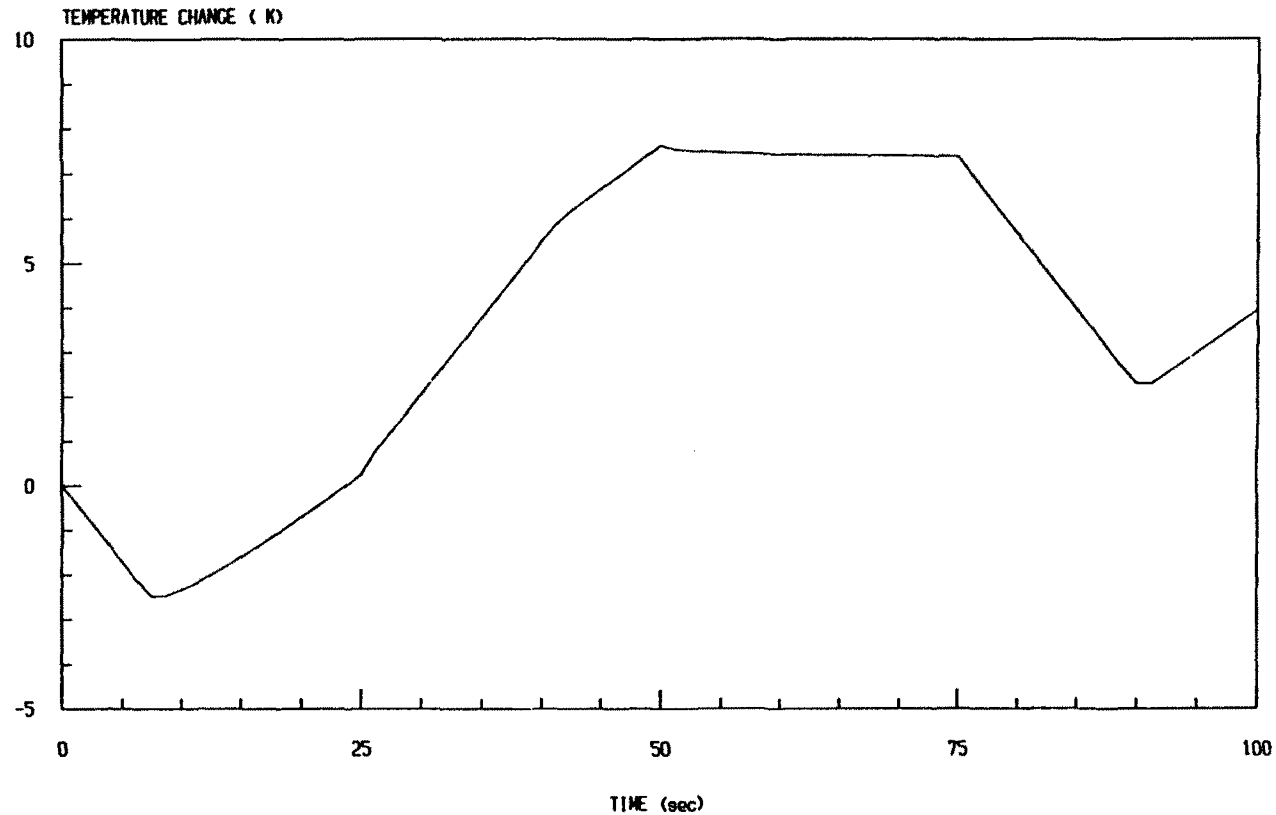

Fig. 9. Predicted temperature change for IN100 at 1005 K (1350°F) subjected to cyclic load history shown in Fig. 8.

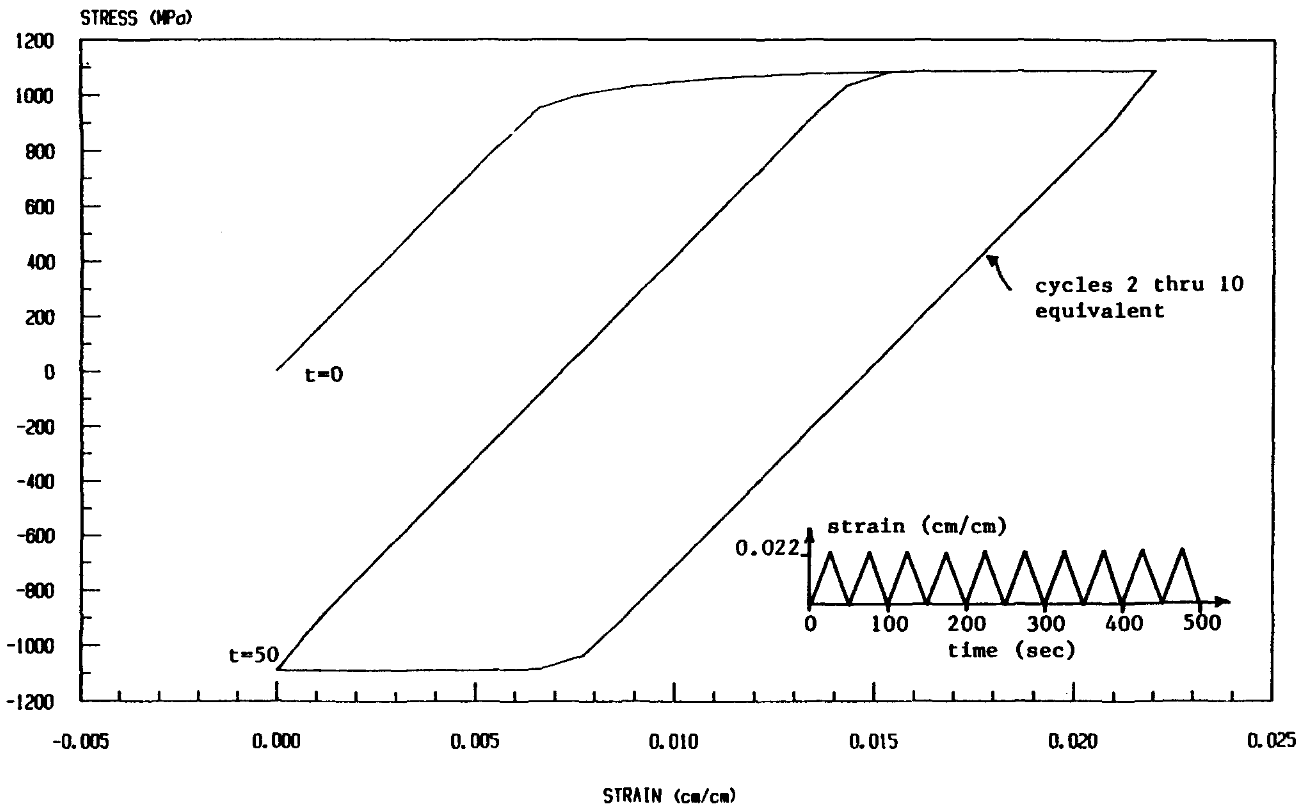

Fig. 10. Predicted stress-strain behavior of IN100 at 1005 K (1350°F) subjected to multi-cyclic load.

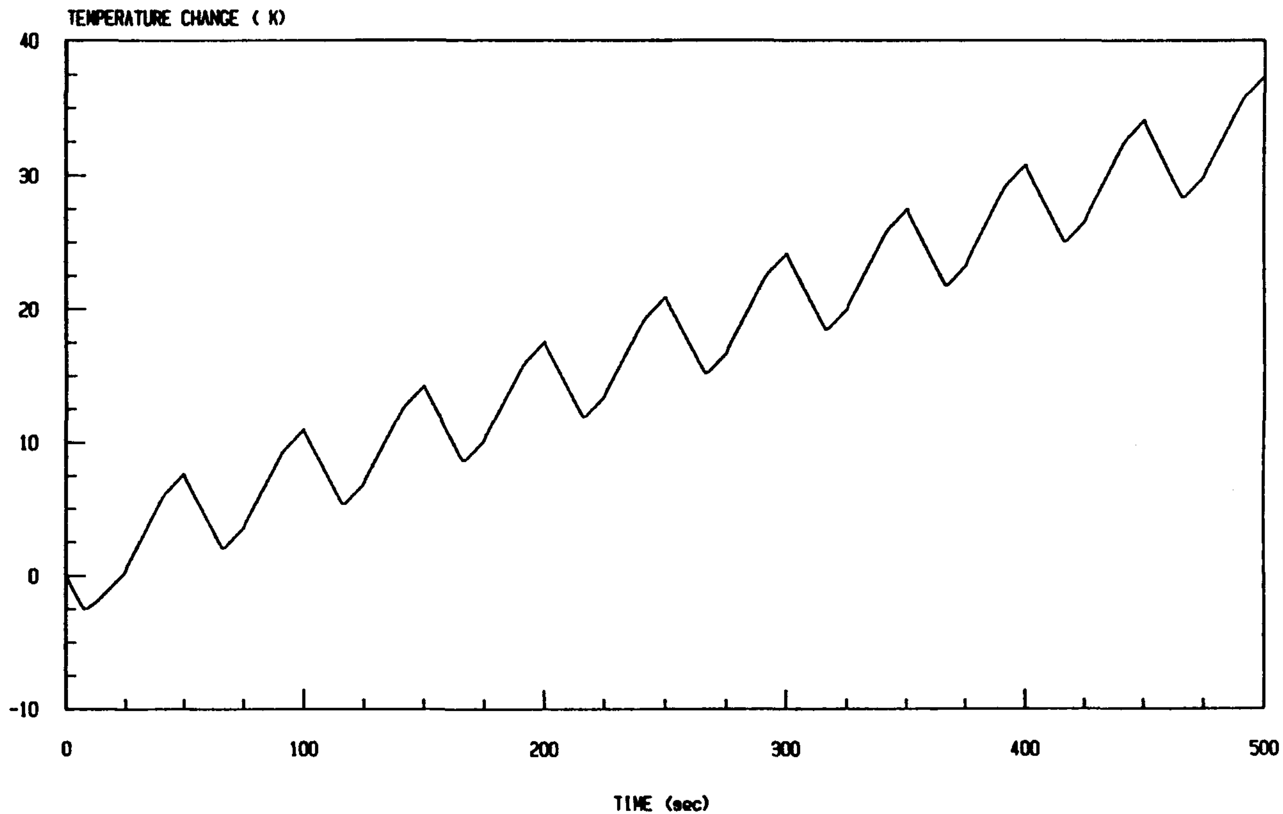

Fig. 11. Predicted temperature change for IN100 at 1005 K (1350°F) subjected to multi-cyclic load history shown in Fig. 10.

obtained by Cernocky and Krempl[11]. This is due to the fact that the assumed internal energy rate described by eqn (14) in [11] can be obtained in the uniaxial form by utilizing eqns (19), (21) and (26) in this article. Further, energy balance equation (55) in [11] can be shown to be identical to eqn (26) derived herein by substituting eqn (32) in this article. Finally, it should be pointed out that under non-adiabatic conditions neglecting the heat flux in the results obtained herein causes increasing overestimation of the temperatures shown in Fig. 3 as the input strain rate decreases.

Bodner and Partom's model[35, 36] may also be written in the uniaxial form described by eqn (32), where

$$\dot{\alpha}_1 = \frac{2}{\sqrt{3}} D_0 \frac{\sigma}{|\sigma|} \exp\left[-\left(\frac{n+1}{2n}\right)\left(\frac{\alpha_2^2}{\sigma^2}\right)^n\right], \tag{37}$$

where D_0 and n are experimentally obtained material constants and

$$\dot{\alpha}_2 = m(Z_1 - \alpha_2)\sigma\dot{\alpha}_1 - AZ_1\left(\frac{\alpha_2 - Z_I}{Z_1}\right)^r, \tag{38}$$

where α_2 is an internal state variable representing drag stress and m, Z_I, Z_1, A and r are experimentally determined material constants. Although eqn (38) contains stress σ, it can be written in the form described by eqn (18) by direct substitution of eqn (32). Thus, Bodner and Partom's model contains two internal state variables in the form described above.

Results are shown in Figs. 4–7 for uniaxial bars of IN 100 pulled at various constant strain rates at an initial temperature of 1005 K (1350°F). Experimental data were obtained from [37], and the material constants described above are tabulated in [38]. The stress-strain curves shown in Fig. 4 are identical to those previously obtained[38].

As a second example using Bodner and Partom's model, a uniaxial bar of IN 100, with material parameters as described in[37] and[38], is subjected to the cyclic strain history shown in Fig. 8 and at initial temperature 1005 K (1350°F). Analytic stress-strain behavior is compared to experiment in Fig. 8 and predicted temperature change is shown in Fig. 9.

Finally, a uniaxial bar is subjected to the multicycle test described in Fig. 10, with resulting temperature rise shown in Fig. 11. It is observed that the model predicts a mean temperature rise of approximately 3.7K (6.7°F) per cycle. The linear increase in mean temperature rise of approximately 3.7 K (6.7°F) per cycle. The linear increase in on the second cycle, which is in agreement with experimental observations at elevated temperature.

CONCLUSION

A model has been presented herein for predicting the temperature rise in uniaxial bars composed of thermoviscoplastic metallic media. The model is also applicable to multiaxial conditions, and this has been reported to some extent in [20]. Although the procedure used here differs from that proposed in [11], it has been shown that the predicted temperature change is identical to results obtained by Cernocky and Krempl when their mechanical constitutive equations are used. However, it has been shown herein that the introduction of internal state variables leads to a more general model which may be used with virtually any thermoviscoplastic model currently used for metals[24].

It has been found in the current research that significant heating may occur under adiabatic conditions, especially during cyclic loading, in thermoviscopolastic metallic media. The significance of this heating is compounded by the fact that material properties often become extremely sensitive in the inelastic range of behavior. This issue has not been considered herein, but it certainly warrants study when transient temperature models become available.

Two important questions have not been answered in this research: (1) what effect does the inclusion of the heat flux term have on the predicted results and (2) what, if anything, does the present model have to do with experimentally observed results? The first question can only be addressed if spacial variation is admitted in the field parameters. The author is currently studying this question and hopes to present results in a future article. The second question cannot be answered at this time since it requires extremely sophisticated experimentation. Although experimental results have been obtained detailing heat generation in inelastic media, it is not possible to compare the current model since additional complex tests must be performed in order to characterize the thermoviscoplastic material parameters. The author also hopes to address this issue in a future article.

Acknowledgement—The author gratefully acknowledges support for this research, which was provided by the Air Force Office of Scientific Research under contract no. F49620-83-C-0067.

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